Illustrating Chemical Concepts with Coin Flipping

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Abstract: Three classroom demonstrations, which involve the participation of students flipping coins, are described. The half-life of a first-order chemical reaction is illustrated by standing students sitting down when they flip a "tails." Dynamic chemical equilibrium is illustrated by students standing or sitting depending on the outcome of coin-flips. In a third demonstration, the motion of gas molecules from one bulb to another is governed by the flip of a coin, showing the improbability of all the molecules gathering in just one bulb.

Students in general chemistry courses need straightforward, everyday examples and analogies to help them understand abstract chemical concepts. Particularly useful are situations that involve their participation. We have found that the simple act of coin flipping helps to illustrate several concepts in chemical kinetics and thermodynamics; some are described below. We have used these demonstrations in classes with as few as 8 and as many as 120 students. The demonstrations may be easily adapted for larger classes. We distribute American one-cent pieces (pennies), but any two-sided fair coin may of course be used.

Half-Life of a First-Order Reaction

The half-life of a first-order reaction is the time it takes for the reactant concentration to decrease to half of its initial value. Mathematically, the fraction of the reactant concentration remaining after n half-lives is given by

$$\left(\frac{1}{2}\right)^n \quad (1)$$

where n = 1, 2, …. A plot of reactant concentration versus number of half-lives, therefore, shows a logarithmic decay curve. To illustrate this concept, everyone in the class is given a coin and asked to stand. On successive commands of the instructor, the students all flip the coins at the same time. Those who end up with heads remain standing while the ones with tails sit down and no longer participate in the coin flipping process. Because each coin flipping corresponds to the elapse of a half-life, a plot of the number of students standing up versus the number of coin flips shows a curve that looks similar to the half-life decay curve. This works particularly well for classes of about 100 students. To save time with larger classes, one should estimate (rather than count) the number of students standing. For small classes, students can be given more than one coin and the instructor repeatedly poll the students for the number of coins that have not yet shown tails.

We have used this demonstration for many years, and we suspect that other teachers have used similar teaching techniques. One of us (JWT) participated in a live demonstration led by Dudley Herschbach at a chemical education conference [1]. By switching the rules in midstream, having all of the standing participants who flip heads (rather than tails) sit down, Herschbach illustrated the point that the choice of tails is arbitrary. In the question and answer period that followed, JWT described the coin-flipping demonstration described below.

Dynamic Equilibrium

Most students understand that equilibrium represents a state with no observable changes in concentrations; however, it is not always obvious to them that the apparently static situation actually involves the balance of the dynamic forward and reverse reactions. Again, coin flipping can help to illustrate both the approach to and the state of dynamic equilibrium.

Initially, the whole class is divided into two groups, with the right-handed students standing and the left-handed students sitting. Each student is supplied with two coins. On the instructor's command, the students will flip the two coins. Those with combinations of HT (heads-tails), TT, and TH are instructed to sit down (or remain seated) and those with HH will stand up (or remain standing). For this "reaction"

$$\text{standing} \leftrightarrow \text{sitting} \quad (2)$$

which is analogous to the chemical reaction

$$\text{reactant} \leftrightarrow \text{product} \quad (3)$$

we have the equilibrium constant

$$K_{eq} = [\text{sitting}] / [\text{standing}] = 3 \quad (4)$$

At the start, the numbers of students standing and sitting represent the initial concentrations of reactants and products. The flipping of the coins corresponds to the forward and reverse reactions. In this analogy, dynamic equilibrium is established after only a single flip of the two coins. Of course, in real systems the time it takes for a reversible chemical reaction to reach dynamic equilibrium may vary greatly, and the equilibrium constant is temperature dependent. The important point to emphasize is that although the overall concentrations do not change (the number of students sitting

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and standing remain the same) at equilibrium, reactants are constantly being converted to products and vice versa (different students will be standing and sitting after each run). This example helps to clarify a common misconception that the equilibrium concentrations are somehow related to stoichiometric coefficients. Here, the equilibrium concentrations (3 reactant:1 product) are clearly different from the stoichiometric coefficients (1 reactant:1 product). An additional concept illustrated here is that the starting conditions do not influence the final equilibrium. In this analogy, one could start with all the students sitting, all the students standing, or any other initial condition, and the same equilibrium state of one quarter of the students standing will be achieved.

Inevitably, in this and other coin demonstrations a student will ask about the case of a coin landing on its edge. Murray and Teare [2] have suggested that the probability of an American nickel (five-cent piece) landing on edge on a flat surface is approximately 1 in 6000 tosses! For the appropriate audience one might draw the analogy of a transition state. If your class is large enough, you might encourage each student to flip coins so that you have tens of thousands of coin toss trials. Please let the authors know what *you* find for the probability of a coin landing on its edge!

Probability, Entropy, and Gas Expansion

The concept of entropy is usually introduced in terms of probability of occurrence. A state of high entropy means a disordered state with a high probability of occurrence while the converse holds true for a state of low entropy. A common example used to calculate an entropy change is to consider the spontaneous isothermal expansion of a gas into a vacuum. Initially, the gas molecules are confined in one bulb, which is separated from a similar but evacuated bulb by a stopcock. Experience tells us that upon opening the stopcock the gas molecules will distribute evenly between the two bulbs, but this even distribution holds only if the number of molecules is very large, say a mole.

To demonstrate the fact that entropy is related only to macroscopic systems, the students are asked to carry out a series of coin tosses and tabulate their results. Each toss represents the location of a molecule. Heads means the molecule ends up in the left bulb while tails means the molecule resides in the right bulb. With the initial 10 or 20 tosses, it is possible to get many more heads than tails, which would mean that most molecules will be found in just one bulb. The probability of finding a molecule in each bulb is one-half, which is the same as the probability of getting a head or a tail from a coin toss. After N tosses (for N molecules), the probability of getting all heads (or finding all the molecules in one bulb) becomes $(1/2)^N$. The quantity $(1/2)^N$ becomes vanishingly small even for relatively small N; say $N = 100$. Repeated coin tossing may be somewhat tedious but the results are of considerable pedagogical value, though we recommend $N > 100$ only for the real skeptics.

Summary and Conclusion

We have found that coin flipping is an effective way of demonstrating several important chemical concepts. It takes relatively little class time and besides its educational merit, this exercise has proven amusing and entertaining for the participants.

References and Notes

1. A Day 2-to-40 Workshop Symposium in Chemical Education. University of Michigan, Ann Arbor, MI, May 1997.
2. Murray, D. B.; Teare, S. W. Probability of a Tossed Coin Landing on Edge. *Phys. Rev. E* **1993,** *48* (4), 2547–2552.